Boson symmetries in vibrational nuclei

Iachello, F; Arima, A

Published in:
Physics Letters B

DOI:
10.1016/0370-2693(74)90389-X

BOSON SYMMETRIES IN VIBRATIONAL NUCLEI

F. IACHELLO and A. ARIMA*
Kernfysisch Versneller Instituut, University of Groningen, Groningen, The Netherlands

Received 25 November 1974

It is proposed that nuclei (away from closed shells and from regions of large deformation) be described as a boson gas. Exploiting the underlying O(5) symmetry analytic expressions for the eigenvalues of the boson Hamiltonian and for the transition matrix elements are obtained. The computed energy levels are in surprisingly good agreement with those recently observed in the (α,$xn\gamma$) and (heavy ions, $xn\gamma$)

The structure of vibrational nuclei has up to now resisted detailed treatment in the framework of both phenomenological and microscopial models. Various semi-empirical expressions have been derived [1] to describe the ground state bands observed in (α,$xn\gamma$) and (heavy ion, $xn\gamma$) experiments, but these expressions fail to simultaneously account for the side bands seen in these nuclei [2, 3]. We have applied a semi-phenomenological version of the interacting boson model of Feshbach and Iachello [4] and found that within this model it is possible to give an unified description of both the ground state and the lateral bands. The picture emerging from our study is that the nucleus (away from closed shells and from regions of large deformation) can to a very good approximation be described as a boson gas. Moreover there appears to be an underlying new and unbroken symmetry. Exploiting the related symmetry group O(5) we have been able to obtain simple analytic expressions for the eigenvalues of the boson Hamiltonian and for the intraband transition matrix elements as well as for side feeding from one band to the other. Backbending occurs naturally as the crossing of two bands and it can be predicted from the relative spacing of the low excited states.

We begin with a list of our basic assumptions which, in the limit of an exact O(5) symmetry, are: (i) collective vibrational levels are built on several quadrupole bosons (hereafter called d-bosons); (ii) the bosons interact with each other and (iii) the interaction V between bosons does not change the boson number. The latter assumption (suggested by detailed microscopic calculations along the lines of ref. [4] is in contrast with previous investigations [5]. It is the origin of the unbroken symmetry. The collective Hamiltonian may then be written as

$$H = \epsilon \sum_m b_m^+ b_m + \sum_\lambda c_\lambda \left[[b^+ b^+]_\mu^\lambda [bb] \right]_0^0, \qquad (1)$$

where

$$c_\lambda = \langle d^2 \lambda\mu | V | d^2 \lambda\mu \rangle, \qquad (2)$$

and $b^+(b)$ is the creation (annihilation) operator for a d-boson. The brackets denote angular momentum couplings.

The nuclear states of N d-bosons are classified by the totally symmetric irreducible representations of the five dimensional orthogonal group O(5). Only five labels are needed to classify the states. Three of them are the total boson number N, the total angular momentum L and its z-component M. The fourth is the seniority v. Instead of v one can introduce another quantum number n_β which counts boson pairs coupled to zero angular momentum. n_β is related to v by

$$v = N - 2n_\beta. \qquad (3)$$

Finally one can introduce a fifth quantum number n_Δ which counts boson triplets coupled to zero angular momentum. The total number N is partitioned by n_β and n_Δ as

$$N = 2n_\beta + 3n_\Delta + \lambda, \qquad (4)$$

and the possible values of the total angular momen-

* Permanent address: Department of Physics, Faculty of Science, University of Tokyo, Tokyo, Japan.

tum L are given in terms of λ by

$$L = \lambda,\ \lambda+1,\ \lambda+2, \ldots\ldots, 2\lambda-2,\ 2\lambda. \tag{5}$$

The absence of the $L = 2\lambda-1$ term is conspicuous.

It is now possible to find the eigenvalues of H within the boson basis Since two d-bosons can only have three values of the total angular momentum, 0, 2 and 4 any interaction V_{ij} between the ith and jth boson can be expressed in terms of three basic operators, the unit operator $\mathbf{1}_{ij}$, the pairing operator P_{ij} and the operator $L_{ij} = 2\boldsymbol{l}_i \cdot \boldsymbol{l}_j$

$$V_{ij} = \alpha \mathbf{1}_{ij} + \beta P_{ij} + \gamma L_{ij} \tag{6}$$

The parameters α, β and γ are related to c_0, c_2 and c_4 by

$$\alpha = \tfrac{1}{14}(6c_4 + 8c_2)$$

$$\beta = \tfrac{1}{10}(c_0 - \alpha + 12\gamma)$$

$$\gamma = \tfrac{1}{14}(c - c_2) \tag{7}$$

The expectation value of H in the state labelled by N, v, n_Δ, L and M does not depend on n_Δ and M. It is given by

$$E(N, v, n_\Delta, L, M) = \epsilon N + \alpha \frac{N(N-1)}{2} + \beta(N-v)(N+v+3) + \gamma[L(L+1) - 6N]. \tag{8}$$

The expectation values of the unit $\mathbf{1}$ and of the L operator used in eq. (8) are trivial. Those of the pairing operator P are obtained by introducing three operators [6]

$$S_+ = \tfrac{1}{2}\sum_m (-)^m b_m^+ b_{-m}^+,\quad S_- = \tfrac{1}{2}\sum_m (-)^m b_m b_{-m}, \tag{9}$$
$$S_0 = \tfrac{1}{4}\sum_m (b_m^+ b_m + b_m b_m^+),$$

in terms of which

$$P = 4\, S_+ S_-. \tag{10}$$

The eigenvalues of the product $S_+ S_-$ can in turn be obtained with the help of the Casimir operator

$$C = S_0^2 - S_0 - S_+ S_-, \tag{11}$$

yielding the result shown in eq. (8). Details of the derivation will be given elsewhere [7].

To the same order of approximation of eq. (1), the quadrupole transition operator Q_m^2 is given by

$$Q_m^2 = q_1(b_m^+ (-)^m b_{-m}) + q_2 [b^+ b]_m^2. \tag{12}$$

Using group theoretical methods similar to those just described one may obtain compact expressions for the matrix elements of Q_m^2 and thus evaluate absolute B(E2) values and branching ratios [7].

The formalism presented here generates spectra with strong regularities. An example is shown in fig. 1; In addition to the splitting of the multiboson states introduced by the boson-boson interaction one observes the appearance of "bands", *a band being defined as a set of levels connected by strong E2 transitions* We have named the important bands Y, X, Z, X$'$, Z$'$, β and Δ and defined them as follows

$$\begin{array}{lll}
\text{Y band} & |N, N, 0, L{=}2N, M\rangle & N{=}0,1,2,\ldots \\
\text{X band} & |N, N, 0, L{=}2N-2, M\rangle & N{=}2,3,4,\ldots \\
\text{Z band} & |N, N, 0, L{=}2N-3, M\rangle & N{=}3,4,5,\ldots \\
\text{X}' \text{ band} & |N, N, 0, L{=}2N-4, M\rangle & N{=}4,5,6,\ldots \\
\text{Z}' \text{ band} & |N, N, 0, L{=}2N-5, M\rangle & N{=}5,6,7,\ldots \\
\beta \text{ band} & |N, N-2, 0, L{=}2N-4, M\rangle & N{=}2,3,4,\ldots \\
\Delta \text{ band} & |N, N, 1, L{=}2N-6, M\rangle & N{=}3,4,5,\ldots
\end{array} \tag{13}$$

Some of these bands are shown in fig. 1, together with the transitions induced by the first part of the operator eq. (12). The second part contributes to the quadrupole moment of the 2_1^+ state and induces transitions between states belonging to the same boson number. These are not shown here for the sake of clarity. Note that the lower parts of the X, Z, X$'$, Z$'$ bands are not very stable. A considerable and increasing fraction of their decays go into the main Y band. This is in agreement with recent experimental results [2, 3] where only the high spin members of the side bands have been observed.

All energies are expressed in terms of four parameters, ϵ, c_0, c_2 and c_4, which can be determined by a fit to the observed energies, Moreover, the Y-band depends only on ϵ and c_4, while the X, Z, X$'$, Z$'$ bands depend only on ϵ, c_4 and c_2. Thus having determined ϵ and c_4 by a fit to the Y-band, one can then use either the X or the Z bands to extract the third parameter c_2 A knowledge of some members of the β-band is however necessary to obtain c_0. Fig. 2 shows an application of eq. (8) to the ^{100}Pd data [2]. Fits of the

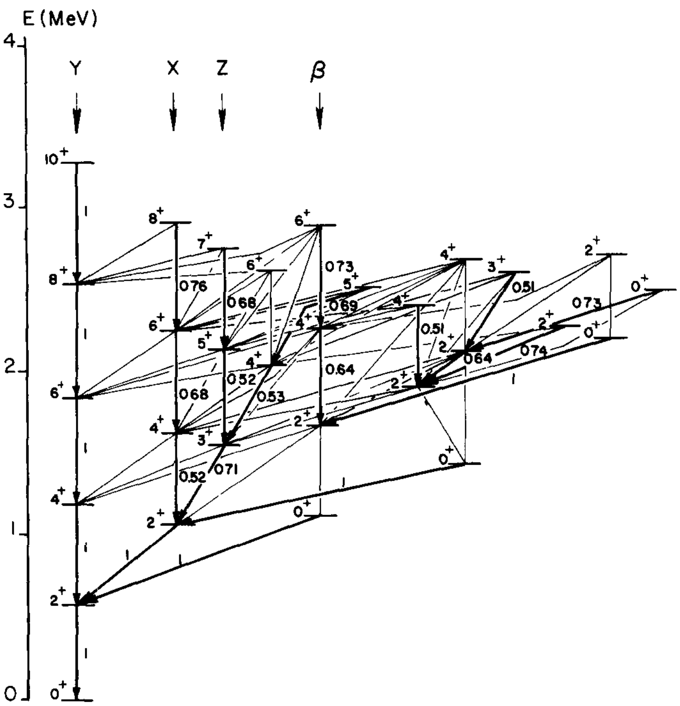

Fig 1. Typical spectrum generated by eq. (8). Strong E2 transitions (branching > 50%) are shown by heavy lines. The numbers next to the line represent branching ratios. The parameters used are ϵ = 579 keV, c_0 = −27.4 keV, c_2 = −95 3 keV, c_4 = 39.4 keV.

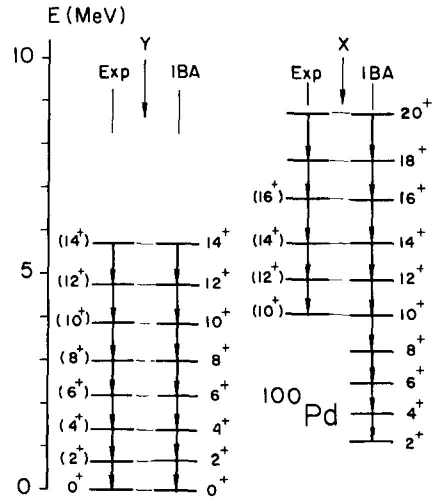

Fig. 2. Comparison between experimental and theoretical (IBA) energies of the Y and X bands in ^{100}Pd. The parameters used are ϵ = 680 keV, c_4 = 45 keV, c_2 = −160 keV. The average deviation between experimental and theoretical energies is 10 keV for the Y-band and 36 keV for the X-band.

same quality have been obtained for ^{102}Pd, ^{154}Dy, ^{148}Sm, ^{152}Gd and ^{156}Er [8].

Although the boson model would appear to be limited to vibrational nuclei we note that the formula for the energies of the members of the Y-band can be rewritten as

$$E(N,N,0,L{=}2N) = \tfrac{1}{8}(4\epsilon - 3c_4)L + \tfrac{1}{8}c_4\,L(L+1), \quad (14)$$

which is identical to the phenomenological formula proposed by Ejiri [9] in his fits to rotational nuclei. This suggests that the boson model may describe transitional and perhaps even some deformed nuclei as well. The possibility that this may be achieved by a simple breaking of the boson symmetry is now being investigated.

Finally we mention that one may easily introduce other degrees of freedom like the octupole boson and single quasi particle fermions. In the case in which the added degree of freedom is another boson the Hamiltonian eq. (1) becomes

$$H = \sum_{lm} \epsilon_l b^\dagger_{lm} b_{lm} + \sum_{\substack{\lambda \\ ll'}} c_{\lambda,ll'} \left[[b^\dagger_l b^\dagger_l]^\lambda_\mu \, [b_{l'} b_{l'}]^\lambda_\mu \right]^0_0 , \quad (15)$$

which is nothing but a shell-model Hamiltonian for bosons.

This work was performed as part of the research programme of the "Stichting voor Fundamenteel Onderzoek der Materie" (FOM) with financial support from the "Nederlandse Organisatie voor Zuiver Wetenschappelijk Onderzoek" (ZWO).

References

[1] M.A.J. Mariscotti, G. Scharff-Goldhaber and B Buck, Phys. Rev. 178 (1969) 1864.
[2] G. Scharff-Goldhaber, M. McKeown, A.H. Lumpkin and W.F. Piel, Phys. Lett. 44B (1973) 416
[3] J.A. Grau, Z.W. Grabowski, F.A. Rickey, P.D. Simms and R.M. Steffen, Phys. Rev. Lett. 32 (1974) 677.

[4] H. Feshbach and F. Iachello, Ann. Phys (N Y.) 84 (1974) 211.
[5] A.K Kerman and C.M. Shakin, Phys. Lett 1 (1962) 151; D.M. Brink, A.F.R. de Toledo Piza and A.K. Kerman, Phys. Lett. 19 (1965) 413; T. Tamura and T. Udagawa, Phys Rev. 150 (1966) 783; B. Sørensen, Phys Lett. 21 (1966) 683; H. Ogata and Y. Akiyama, Phys. Lett. B29 (1969) 558; D.R Bes and G.C. Dussel, Nucl. Phys. A135 (1969) 1; T.K. Das, R.M. Driezler and A. Klein, Phys. Rev. C2 (1970) 632; G. Gneuss, V. Mosel and W. Greiner, Phys. Lett. 30B (1969) 397; 31B (1970) 209; 32B (1970) 161; G Gneuss and W Greiner, Nucl. Phys. A171 (1971) 449
[6] H J. Lipkin, Lie groups for pedestrians (North-Holland Publ. Co., Amsterdam 1965).
[7] A Arima and F Iachello, paper in preparation.
[8] F. Iachello, Collective aspects of the shell-model, invited talk at the Int. Conf. on Nucl. Structure and Spectroscopy, Amsterdam, Sept. 1974
[9] H. Ejiri, Institute for Nuclear Studies (Tokyo) Rep. Nos. INSJ 101 (1966) and 104 (1967), unpublished; H. Ejiri, M. Ishikara, M. Sakai, K. Katori and T. Imamura, J. Phys. Soc. (Japan) 24 (1968) 1189